OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4729

Mechanics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4729
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

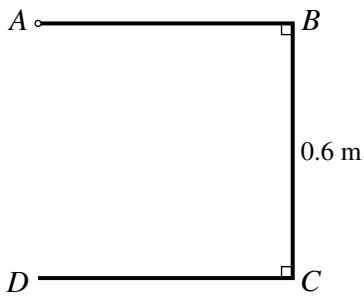
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1



A uniform square frame $ABCD$ has sides of length 0.6 m. The side AD is removed from the frame, and the open frame $ABCD$ is attached at A to a fixed point (see diagram).

(i) Calculate the distance of the centre of mass of the open frame from A. **[5]**

The open frame rotates about A in the plane $ABCD$ with angular speed $3\,\text{rad}\,\text{s}^{-1}$.

(ii) Calculate the speed of the centre of mass of the open frame. **[2]**

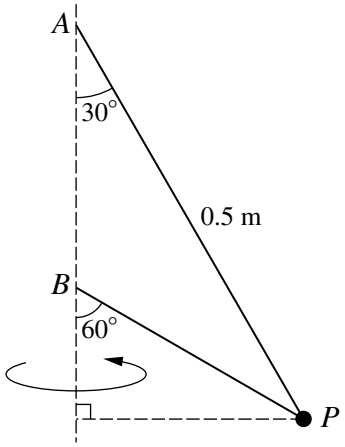
2 The resistance to the motion of a car is $kv^{\frac{3}{2}}$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the car's speed and k is a constant. The power exerted by the car's engine is 15 000 W, and the car has constant speed $25\,\text{m}\,\text{s}^{-1}$ along a horizontal road.

(i) Show that $k = 4.8$. **[3]**

With the engine operating at a much lower power, the car descends a hill of inclination α, where $\sin\alpha = \frac{1}{15}$. At an instant when the speed of the car is $16\,\text{m}\,\text{s}^{-1}$, its acceleration is $0.3\,\text{m}\,\text{s}^{-2}$.

(ii) Given that the mass of the car is 700 kg, calculate the power of the engine. **[5]**

3



A particle P of mass 0.4 kg is attached to one end of each of two light inextensible strings which are both taut. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The string AP makes an angle of 30° with the vertical and is 0.5 m long. The string BP makes an angle of 60° with the vertical. P moves with constant angular speed in a horizontal circle with centre vertically below B (see diagram). The tension in the string AP is twice the tension in the string BP. Calculate

(i) the tension in each string, **[4]**

(ii) the angular speed of P. **[4]**

4 A block of mass 25 kg is dragged 30 m up a slope inclined at 5° to the horizontal by a rope inclined at 20° to the slope. The tension in the rope is 100 N and the resistance to the motion of the block is 70 N. The block is initially at rest. Calculate

(i) the work done by the tension in the rope, **[2]**

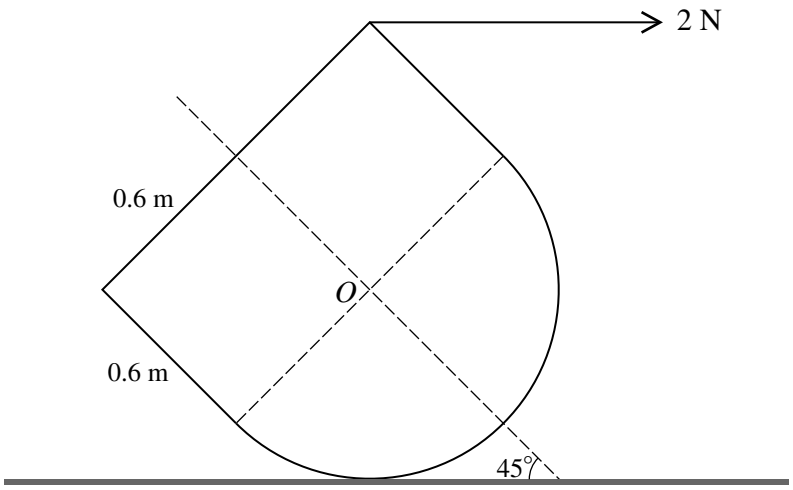
(ii) the change in the potential energy of the block, **[2]**

(iii) the speed of the block after it has moved 30 m up the slope. **[4]**

5 A uniform solid is made of a hemisphere with centre O and radius 0.6 m, and a cylinder of radius 0.6 m and height 0.6 m. The plane face of the hemisphere and a plane face of the cylinder coincide. (The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.)

(i) Show that the distance of the centre of mass of the solid from O is 0.09 m. **[5]**

(ii)



The solid is placed with the curved surface of the hemisphere on a rough horizontal surface and the axis inclined at 45° to the horizontal. The equilibrium of the solid is maintained by a horizontal force of 2 N applied to the highest point on the circumference of its plane face (see diagram). Calculate

(a) the mass of the solid, **[4]**

(b) the set of possible values of the coefficient of friction between the surface and the solid. **[3]**

[Questions 6 and 7 are printed overleaf.]

6 A small ball B is projected with speed $14\,\mathrm{m\,s^{-1}}$ at an angle of elevation 30° from a point O on a horizontal plane, and moves freely under gravity.

(i) Calculate the height of B above the plane when moving horizontally. **[2]**

B has mass 0.4 kg. At the instant when B is moving horizontally it receives an impulse of magnitude I N s in its direction of motion which immediately increases the speed of B to $15\,\mathrm{m\,s^{-1}}$.

(ii) Calculate I. **[3]**

For the instant when B returns to the plane, calculate

(iii) the speed and direction of motion of B, **[4]**

(iv) the time of flight, and the distance of B from O. **[5]**

7 Three small smooth spheres A, B and C of masses 0.2 kg, 0.7 kg and m kg respectively are free to move in a straight line on a smooth horizontal table. Initially B and C are stationary and A is moving with velocity $1.8\,\mathrm{m\,s^{-1}}$ directly towards B. The coefficient of restitution for the collision between A and B is e. Immediately after this collision the speed of A is greater than the speed of B.

(i) Calculate the set of possible values of e. **[9]**

It is now given that the speed of B immediately after the collision with A is $0.75\,\mathrm{m\,s^{-1}}$. B continues its motion and strikes C directly in a perfectly elastic collision. B has speed $0.25\,\mathrm{m\,s^{-1}}$ immediately after its collision with C.

(ii) Calculate the two possible values of m. **[6]**

Question			Expected Answer	Mark	Rationale/Additional Guidance
1	(i)		$3x_G = 2x0.3 + 1x0.6$ OR $3x_G = 2x0.3 + 0$ OR $3x_G = 4x0.3$ OR $3y_G = 1x0.3 + 1x0.6 + 0$ OR $3y_G = 4x0.3 - 1x0.3$ $x_G = 0.4$ (from AD) OR $x_G = 0.2$ (from BC) $y_G = 0.3$m from AB or CD $AG^2 = 0.4^2 + 0.3^2$ AG = 0.5 m	M1 A1 A1 M1 A1 **[5]**	Table of moments idea. M0 for reducing to 1D problem. Masses/weights may be included. Pythagoras with 2 appropriate distances. This may only be seen in (ii), allow M1A1 in this case.
	(ii)		v = 0.5x3 v = 1.5 ms^{-1}	M1 A1 **[2]**	Allow use of candidate's 0.2, 0.4, 0.3, 0.5
2	(i)		$(k25^{3/2}) \times 25 = 15000$ k = 4.8 **AG**	M1 A1 A1 **[3]**	Tractive force x speed = power
	(ii)		$R = 4.8 \times 16^{3/2}$ $T - 4.8x16^{3/2} + 700gx1/15 = 700x0.3$ P = 59.9 x 16 P = 958 W	B1 M1 A1 M1 A1 **[5]**	307.2 N2L, 4 terms to find tractive force (T) Allow cv(R), R not 600; (T = 59.866..) 16xTractive force

3	**(i)**		$T_A\cos30 + T_B\cos60 = 0.4g$ $2T\cos30 + T\cos60 = 0.4g$ $T_B = 1.76$ N $T_A = 3.51$ N	M1 A1 A1 A1 **[4]**	Resolves vertically, 3 terms T = 1.756. Watch for MR of Tcos30 + 2Tcos60 = 0.4g Accept 3.52
	(ii)		r = 0.5sin30 (= 0.25) $3.51\sin30 + 1.76\sin60 = 0.4\omega^2 0.5\sin30$ $\omega = 5.72$ rad s^{-1}	B1 M1 A1ft A1 **[4]**	 N2L radial, 3 terms cv(1.76, 3.51, 0.25) Accept 5.73
4	**(i)**		WD = 100cos20 x 30 WD = 2820 J	M1 A1 **[2]**	Product of 3 relevant elements. Angle could be 5, 25 or complements 2819.1...
	(ii)		PE = 25g x 30sin5 PE = 641	M1 A1 **[2]**	Product of weight and vertical height. Allow without g 640.6
	(iii)		 2819.1 = 640.6 + 30x70 + $25v^2/2$ v = 2.51 ms^{-1}	M1 A1ft A1 A1 **[4]**	4 term energy equation ft(cv 2820 and cv 641) cao
		OR	25a = 100cos20 -70 -25gsin5 a = 0.105 $v^2 = 2 \times 30 \times$ 'a' v = 2.51	*M1 A1 dep*M1 A1 **[4]**	4 term equation Allow 0.1 here Or equivalent complete method cao

5	**(i)**		$x_H = 3x0.6/8$ $\pi(0.6^2x0.6)(0.6/2) - (0.6^3x2\pi/3)0.225$ $= \pi x0.6^3(1+2/3)x_G$ $x_G = 0.09$ m **AG**	B1 M1 A1 A1 A1 **[5]**	CoM hemisphere ($x_H = 0.225$), may be implied Use of table of moments idea SC Volume of sphere used, max B1M1A1, moment equation fully correct for A1 (3/5) Accept -0.09
	(ii)	**(a)**	 $mg(0.09\cos45) =$ $2(0.6+0.6\cos45+0.6\sin45)$ $m = 4.65$kg	M1 A1 A1 A1 **[4]**	Attempt at moments (must resolve), allow without g $2(0.6+\sqrt{[0.6^2+0.6^2]})$ (4.6451...)
	(ii)	**(b)**	 $2/4.6451g$ $\mu \geq 0.0439$	M1 A1 A1 **[3]**	Ratio force/weight cv(4.65) Correct inequality sign, accept 0.044
6	**(i)**		$0 = (14\sin30)^2 - 2gh$ $h = 2.5$ m	M1 A1 **[2]**	$h = (14\sin30)x1/1.4 - g(1/1.4)^2/2$ or use $(u^2\sin^2\theta)/2g$
	(ii)		 $0.4x15 = 0.4(14\cos30) + I$ $I = 1.15$	M1 A1 A1 **[3]**	Impulse = change in momentum Not 14 or 0 for horizontal speed before impulse aef
	(iii)		$v^2 = (14\sin30)^2 + 15^2$ $v = 16.6$ ms^{-1} $\tan\theta = 14\sin30/15$ OR $\tan\psi = 15/14\sin30$ $\theta = 25(.0)°$ OR $\psi = 65(.0)°$	M1 A1 M1 A1 **[4]**	Not $(14\sin30)^2 + (14\cos30)^2$ Allow $\sqrt{274}$ Correct trig to find an appropriate angle; not 14cos 30 for 15
	(iv)		$t = 14\sin30/g$ (= 1/1.4 = 0.7142..) $T = 1.43$ s $R = 14\cos30/1.4 + 15/1.4$ $R = 19.4$ m	M1 A1 M1A1 A1 **[5]**	Rise or fall time (not to be given in (i)) Accept 10/7 $(14^2\sin(2x30) + 16.6^2\sin(2x25))/2g$. 14 resolved, 15 not

7	(i)			M1	Uses restitution
			b + a =1.8e	A1	b - a =1.8e
				M1	Uses momentum
			0.7b - 0.2a=0.2x1.8	A1	0.7b + 0.2a=0.2x1.8, signs consistent with first eqn
				M1	Solves 2 simultaneous equations (eliminate a or b)
			b =0.4(1+e)	A1	
			a = 1.4e - 0.4	A1	a = 0.4 - 1.4e
			1.4e - 0.4 > 0.4 + 0.4e	M1	Using a>b, correct signs in a essential
			e > 0.8	A1	
				[9]	
	OR	**Last 5 marks**	Using a > b	M1	correct signs in a essential
			a > 0.72	A1	
			b > 0.72	A1	
			1.8e > 0.72 + 0.72	M1	
			e > 0.8	A1	
	OR	**Last 5 marks**	Using a = b to find a or b	M1	
			a (or b) = 0.9e and a (or b) = 0.72	A1	
			e = 0.8	A1	
			Convincing argument for correct inequality	M1	
			e > 0.8	A1	
	OR	**Last 5 marks**		M1	Solves 2 simultaneous equations (eliminate a or b)
			a = 1.4e - 0.4 or b =0.4(1+e)	A1	aef or multiples thereof
			Using a > b	M1	correct signs in a essential
			a > 0.9e or b < 0.9e	A1	aef or multiples thereof
			e > 0.8	A1	

	(ii)		c – (±0.25) = 1x0.75 c = 0.5, 1 0.75x0.7 = 0.25x0.7 + m (x1) *OR* 0.75x0.7 = -0.25x0.7 + 0.5m m = 0.35 (from first equation) m=1.4 (from second equation)	M1 A1A1 M1 A1 A1 **[6]**	Uses restitution with e = 1, either Or 0.75 ± 0.25 Uses momentum conservation with correct combination of sign and c value *OR* mx(0.75 ± 0.25) ± 0.7x0.25 = 0.75x0.7
	OR		½ x0.7x0.75^2 = ½ x0.7X0.25^2 + ½mc^2 0.7x 0.75 = 0.7x(+/-0.25) + mc Solving simultaneous equations m = 0.35 m = 1.4	B1 M1 A1 M1 A1 A1	½ may not be seen At least one momentum equation mc = 0.35 and 0.7
			Total	**[72]**	

[END]